Van Fraassen and Ruetsche on Preparation and Measurement

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Abstract

Ruetsche (1996) has argued that van Fraassen's (1991) Copenhagen Variant of the Modal Interpretation (CVMI) gives unsatisfactory accounts of measurement and of state preparation. I defend the CVMI against Ruetsche's first argument by using decoherence to show that the CVMI does not need to account for the measurement scenario which Ruetsche poses. I then show, however, that there is a problem concerning preparation, and the problem is more serious than the one Ruetsche focuses on. The CVMI makes no substantive predictions for the everyday processes we take to be measurements.

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1. Introduction. The main intuitive motivation for Bas van Fraassen's (1991) Copenhagen Variant of the Modal Interpretation (CVMI) is to give an interpretation of quantum mechanics that is empirically adequate, includes measuring devices in the quantum-mechanical dynamics, and remains as faithful as possible to the desiderata of the original Copenhagen interpretation. The four main desiderata are that all quantum-mechanical description is given in terms of state attributions, no value-attributing propositions can be true together unless they can be certain together, a system is assigned a particular definite value state only when required, and the eigenstate-eigenvalue link (van Fraassen, 1991, pp. 241, 314, 280, 247). In his CVMI, van Fraassen subscribes to all the desiderata but the last: he rejects the eigenstate-eigenvalue link in favor of the eigenstate-to-eigenvalue half-link. The CVMI specifies that an observable of a system can have a definite value even when the system is not in an eigenstate of that observable.

To understand the rules for attribution of value states in the CVMI, we'll first examine the attribution of value states in the Copenhagen interpretation. According to the eigenstate-eigenvalue link, a system is in an eigenstate of some observable iff the observable has the value associated with that eigenstate. When the system is not in an eigenstate, the observable has no definite value. The eigenstate-eigenvalue link is incompatible with the quantum-mechanical dynamics and the experience we have of measurement outcomes; this inconsistency is (one version of) the measurement problem.

To see this, consider a standard measurement situation, with object system S, object observable **O**, eigenbasis $\{|o_i\rangle\}$, measurement apparatus system R, apparatus observable **P**, eigenbasis $\{|p_i\rangle\}$, and ready state $|p0\rangle$. (Here as elsewhere I follow Ruetsche's notation.) If the pre-measurement object state is a superposition of **O** eigenstates, an ideal measurement of **O** by **P** evolves according to the quantum-mechanical dynamics as follows:

$$\sum_i c_i |o_i\rangle \otimes |p0\rangle \rightarrow \sum_i c_i |o_i\rangle \otimes |p_i\rangle = |\Psi^{SR}\rangle.$$

The reduced apparatus state is given by tracing over the degrees of freedom of S:

$$\mathbf{W}^R = \mathrm{Tr}_S(|\Psi^{SR}\rangle\langle\Psi^{SR}|) = \sum_i |c_i|^2 |p_i\rangle\langle p_i|.$$

Since $\mathbf{W}^R$ is not a **P** eigenstate, by the eigenstate-eigenvalue link **P** has no definite value; this contradicts our experience that a measuring apparatus does have a definite value at the end of measurement.

The Copenhagen interpretation thus faces a measurement problem. Von Neumann, for example, attempts to solve the problem by changing the dynamics, specifying that the apparatus state collapses into a **P** eigenstate when the measurement occurs. The CVMI attempts to solve the problem by giving up the eigenstate-eigenvalue link, allowing **P** to have a definite value at the end of a measurement even when $\mathbf{W}^R$ is not a **P** eigenstate.

2. Measurement. The CVMI specifies what determinate values the observables of a system has in two circumstances: when the system is in an eigenstate of some observable, and when the system is at the end of being involved in a measurement. An important virtue of the CVMI is that it gives an account of measurement in purely quantum-mechanical terms. I will now present the conditions that an evolving system must satisfy in order to be considered a measurement process.

A measurement process, according to the CVMI, must satisfy two conditions (van Fraassen 1991, 211-213). The first is the probability reproducibility condition, which specifies that, for any initial object state $|\psi\rangle$,

(M1) $$\mathrm{Tr}(|\psi\rangle\langle\psi|o_i\rangle\langle o_i|) = \mathrm{Tr}(\mathbf{W}^R |p_i\rangle\langle p_i|).$$

Roughly, according to this condition each pointer eigenvalue must have the same probability of obtaining after the measurement as the corresponding object eigenvalue had before the measurement.

The second condition a measurement process must satisfy is that, for any initial object state, the post-measurement reduced apparatus state must be a mixture of **P** eigenstates; that is,

(M2) $$\mathbf{W}^R = \sum_i w_i |p_i\rangle\langle p_i|.$$

According to the CVMI, an interaction between an object initially in state $|\psi\rangle$ and an apparatus initially in state $|p0\rangle$ is a measurement process iff (M1) and (M2) are satisfied.[1]

Now that we have specified what measurements are according to the CVMI, we can state the rules which determine what values observables of systems have. The CVMI specifies four rules:

(a) The eigenstate-to-eigenvalue half-link: when a system is in an eigenstate of an observable, the observable has the value corresponding to that eigenstate. (van Fraassen 1991, pp. 281, 313)

(b) If a system is in the state $\sum_i c_i |q_i\rangle \otimes |p_i\rangle$, where $\{|q_i\rangle\}$ is an orthogonal set of unit vectors, and that state is the result of an **O** measurement by pointer-observable **P**, then the probability is $|c_k|^2$ that **P** has value p_k. (van Fraassen 1991, 289)

(c) If a system is in the state $\sum_i c_i |o_i\rangle \otimes |p_i\rangle$, and that state is the result of an **O** measurement by pointer-observable **P**, then the probability is $|c_k|^2$ that both **P** has value p_k and **O** has value o_k. (van Fraassen 1991, pp. 287, 324)

(d) When none of the above rules apply, the system has an unspecified value state *V*; the only

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[1] In the next section I will consider the more general case where the initial apparatus state isn't pure.

constraint is that there exists a quantum state in the image space of $\mathbf{W}^{SR}$ for which the value state V has probability 1 of obtaining. (van Fraassen 1991, pp. 281, 307)

Laura Ruetsche (1996) argues that there exists a class of interactions which are typically considered to be measurements, but which do not fulfill van Fraassen's criteria for measurements. These interactions, called General Unitary Measurements (GUMs), are of the form

(GUM) $\sum_i c_i |o_i\rangle \otimes |p0\rangle \rightarrow \sum_i c_i |q_i\rangle \otimes |p_i\rangle$
where the $\{|q_i\rangle\}$ are a non-orthogonal set of unit vectors.

GUMs fulfill (M1), but not (M2). Since the $\{|q_i\rangle\}$ are non-orthogonal, the reduced apparatus state $\mathbf{W}^R$ will be a mixture of states which are not **P** eigenstates. As a consequence, GUMs are not measurements according to the CVMI, and thus rules (b) and (c) cannot be applied. Hence, the CVMI does not specify probabilities for the value of **P** at the end of a GUM interaction.

Ruetsche takes this to be a problem for the CVMI. She writes that "By van Fraassen's lights, GUMs will not have outcomes. Neglecting GUMs, van Fraassen fails to appreciate how the Measurement Problem might recur in" the CVMI (Ruetsche 1996, S342).

While it is true that the CVMI does not guarantee that **P** has a value at the end of a GUM, this is not a problem. It would be a problem were GUMs to actually occur in our interactions with systems; if this were the case, then the CVMI would be radically incomplete, because it would not predict even probabilities for measurement outcomes we experience. Ruetsche does not point out, however, that GUMs never occur in the experiments that we are capable of performing.

Why is it that GUMs never occur in the experiments that we are capable of performing? Suppose that R is a microscopic apparatus; for us to observe the measurement outcome, R must interact with some larger system which is capable of presenting the result of R's measurement.

Suppose that R is a macroscopic apparatus; we know from the literature on environmental decoherence that R will interact with its environment, or that a part of R can be treated as the environment for the rest of R. (For a summary see Bub 1997, 155-163.) Thus, where E is the environment of R, $\{|e_i\rangle\}$ a set of orthogonal states of E, and $|e0\rangle$ the initial state of E, a better model of processes that actually occur is

(GUME) $\sum_i c_i |o_i\rangle \otimes |p0\rangle \otimes |e0\rangle \rightarrow \sum_i c_i |q_i\rangle \otimes |p_i\rangle \otimes |e_i\rangle$
where the $\{|q_i\rangle\}$ are a non-orthogonal set of unit vectors.

GUMEs (Generalized Unitary Measurements with Environment) *do* fulfill the CVMI's two criteria for measurements. Condition (M1) is fulfilled because the probability distribution for $\mathbf{O}\otimes\mathbf{I}^{\mathbf{E}}$ is transcribed to the probability distribution for **P**. ($\mathbf{O}\otimes\mathbf{I}^{\mathbf{E}}$ is the new object observable; $\mathbf{I}^{\mathbf{E}}$ is the identity operator on E's Hilbert space.) Condition (M2) is fulfilled because when one traces over the state of S+E, the reduced apparatus state is diagonal in the **P** eigenbasis. The reason (M2) holds for GUMEs but not GUMs is that for GUMEs, the set of states one traces over to generate the reduced apparatus state are orthogonal, while for GUMs they are not.

So GUMEs are measurements according to the CVMI. We now can apply rule (b) to show that the CVMI specifies the appropriate probability distribution over the values of **P**. Rule (b) is written for two-body systems, so let us consider the system of (S+E) and R. At the end of a GUME, we have a system in state $\sum_i c_i (|q_i\rangle\otimes|e_i\rangle) \otimes |p_i\rangle$, and that state is the result of an $\mathbf{O}\otimes\mathbf{I}^{\mathbf{E}}$ measurement by pointer-observable **P**, so by rule (b) the probability is $|c_k|^2$ that **P** has value p_k. Thus, the CVMI does specify the appropriate probability distribution for the values of **P**. I conclude that Ruetsche's GUM argument is unsuccessful.

There is an idealization that needs to be removed, but I haven't done so before now because removal complicates matters. Any time a real-world measurement is made, it's actually the case that the relevant environment states $\{|e_i\rangle\}$ are *non-orthogonal*; the states are just very close to orthogonal. Application of the biorthonormal decomposition theorem leads to the result that the final reduced apparatus state is a mixture of $\mathbf{P}^{\#}$ eigenstates, not **P** eigenstates, where $\mathbf{P}^{\#}$ is very close to **P** as long as the $|c_i|$'s are not close to one another. $\mathbf{P}^{\#}$ is close to **P** in the sense that the eigenbasis vectors of $\mathbf{P}^{\#}$ are very close to the eigenbasis vectors of **P**: $\langle p_i^{\#}|p_i\rangle \simeq 1$. It is commonly believed that, in all the measurement situations people have been in, the $|c_i|$'s have not been close enough for this difference between $\mathbf{P}^{\#}$ and **P** to be detectable.

Nevertheless, there is a problem here for the CVMI. The probability reproducibility condition (M1) does not hold for non-idealized GUMEs. Just as the vectors in the eigenbasis of $\mathbf{P}^{\#}$ are only very close to the vectors in the eigenbasis of **P**, $\mathrm{Tr}(|\psi\rangle\langle\psi|o_i\rangle\langle o_i|)$ is only very close to $\mathrm{Tr}(\mathbf{W}^{R}\,|p_i^{\#}\rangle\langle p_i^{\#}|)$. Since condition (M1) is not met, a measurement has not occurred, so rules (b) and (c) cannot be applied. This explains David Albert's *en passant* criticism of the CVMI, which I quote in full:

> But the trouble (here as before) is that none of the (imperfect) measurements which we actually *carry out* will ever precisely *satisfy* van Fraassen's characterization. And so there isn't ever going to be a matter of fact (in the real world) about what observable the record observable *is*. And so van Fraassen's algorithm will in general pick out *nothing whatsoever* (over and above what gets picked out by [the basic principles of quantum mechanics]) as well-defined. (1992, 197)

Van Fraassen has two possible responses to this line of argument: he can claim that his interpretation is only meant to apply in ideal cases, or he can modify his interpretation so that the criticism no longer applies. Van Fraassen discusses both these options in his book; while he embraces the first, I shall argue for the second.

(1) Van Fraassen points out with regard to (M1) that "We are idealizing to the extent of asking the final apparatus state to reproduce the relevant statistics exactly" (1991, 225). Further, he recognizes that this position is open to the above objection: "with a strict criterion for measurement, we end up with *no* predictions for the processes we usually refer to as measurement" (1991, 232). To get around this problem, van Fraassen takes an "empiricist view" and suggests that acceptance of a theory like the CVMI "involves the decision to let the theory function as expert predictor (probability assigner) for the phenomena as *we* classify them" (1991, 233). Here the idea is that we can take the real-world phenomena, classify them in an ideal way, and then use the theory to make idealized predictions. So for the situation that we've been considering, we can take a GUME, specify that the environment states $\{|e_i\rangle\}$ *are* orthogonal, and then use the CVMI to make predictions for this idealized situation.

As an aspiring empiricist, I reject this purported application of empiricism, and van Fraassen should too. The empiricist recognizes the importance of having a theory with which one can make predictions, even if the predictions are idealized. However, the empiricist wants more than that out of a theory. According to van Fraassen's constructive empiricism, when an empiricist accepts a theory, she believes that the theory is *empirically adequate* (van Fraassen 1980, 12). In *The Scientific Image* (p. 64) and elsewhere, van Fraassen requires that an empirically adequate theory has a model with an empirical substructure which *directly* represents all observable phenomena in the domain of the theory. Thus the CVMI is empirically adequate only in a trivial sense, since (except for the eigenstate-to-eigenvalue half-link) the theory makes "*no* predictions for the processes we usually refer to as measurement" – the theory only makes predictions for *idealized* processes. The CVMI lacks the pragmatic virtues that the empiricist desires, and hence does not deserve acceptance.

(2) A modified version of the CVMI is, however, potentially worthy of acceptance. As an alternative to (M1), I propose the condition that for any initial object state $|\psi\rangle$, and for some object observable **O** with eigenbasis $\{|o_i\rangle\}$,

(M1*) $$\sum_i |\mathrm{Tr}(|\psi\rangle\langle\psi|o_i\rangle\langle o_i|) - \mathrm{Tr}(\mathbf{W}^R |p_i^{\#}\rangle\langle p_i^{\#}|)| \leq \epsilon,$$

where ϵ is a new fundamental constant, and is taken to be some small number. This new theory allows for the processes that we refer to as measurements to really be measurements. The pointer observable $\mathbf{P}^{\#}$ picked out by (M2) is close to the anticipated pointer observable **P**, and hence the pre-measurement probabilities assigned to eigenvalues of **O** are close to the post-measurement probabilities assigned to the eigenvalues of $\mathbf{P}^{\#}$.

Van Fraassen rejects the option of making the criteria for measurement more permissive, because he believes that if we do so “we shall imply that incompatible observables can be jointly measured” (1991, 232); that is, there are multiple incompatible object observables which can be taken to be **O**. But this problem can be dissolved by specifying that, for measurements where (M1*) but not (M1) holds, rule (c) for determining what values observables of systems have does not apply. For such measurements, the value state for observable **O** will never be specified, and so it doesn’t matter that there are multiple observables which can be taken to be **O**. This modification doesn’t affect the empirical adequacy of the CVMI: what’s important for empirical adequacy is that $\mathbf{P}^{\#}$ has a determinate value at the end of the measurement, and that the values for $\mathbf{P}^{\#}$ have the appropriate probabilities of obtaining. This is guaranteed by rule (b).

To sum up: Ruetsche criticizes the CVMI for having overly restrictive conditions on what counts as a measurement; she argues that the CVMI cannot account for GUMs. I agree, but point out that none the measurements we're capable of performing are GUMs. GUMEs are a better model of our measurement practices, and the CVMI can almost account for them. I defend a modification of the CVMI which allows it to predict the results of measurements like GUMEs.

3. Preparation. The CVMI[2] requires that in a measurement situation the apparatus system starts out in a ready state. Van Fraassen writes:

> The most general notion of measurement requires therefore only that we be able to infer from information of outcomes to information about the measured object system's initial state. This may be formally captured as follows. A measurement process of observable *A* on object system *X* is characterized by four factors: the Hilbert space of the measuring apparatus *Y*, the pointer-observable *B*, the groundstate *W* of the apparatus, and the evolution operator *U*. ... The initial reduced state of the apparatus must be the groundstate *W*. (1991, 211)

Van Fraassen goes on to explain that the apparatus state *W* evolves via *U*, and the resulting final state must fulfill condition (M1).

Nowhere does van Fraassen discharge the assumption that the apparatus starts in a ready state; indeed he calls this account "the most general notion of measurement". It's important to note that the state *W* is a quantum, or "dynamic", state, not a value state.

This requirement that the apparatus starts out in a ready state is not compatible with our experimental practice of pre-measurement preparation. Suppose one wants to make an x-spin measurement with apparatus R on a z-spin up electron. A typical way to prepare such an electron would be to take some unprepared electrons and pass them through a Stern-Gerlach device with magnetic field oriented along the z-axis. Electrons deflected in the upwards direction are z-spin up

[2] In this section I am discussing both the original CVMI and my modified version proposed above.

electrons, and can then interact with R. Suppose that, before the electrons interact with R, a non-demolition position measurement is made on the electron to discover whether it was deflected up or down. Suppose further that if the electron is deflected down, R is made to evolve from its ready state to a non-ready state: for example, a bomb could explode, scattering the particles that make up R. If the particle is deflected up, it is allowed to interact with R without incident.

The problem for the CVMI is that just before the electron interacts with R, the state of R is not a ready state, and hence the interaction of R with the electron does not count as a measurement. Just before interaction the system is in the state (idealized by leaving out the environment):

$$a\,|\text{z-spin up}\rangle_e \otimes |\text{ready}\rangle_R \;+\; b\,|\text{z-spin down}\rangle_e \otimes |\text{non-ready}\rangle_R\,,$$

where $a,b \neq 0$ and $|a|^2 + |b|^2 = 1$. Thus, the state of R is a mixture of a ready state and a non-ready state, and the resulting interaction doesn't count as a measurement.

One might wonder why the CVMI doesn't attempt to get around this problem by requiring only that the *value* state of the apparatus be a ready state. But to fulfill the Copenhagen desiderata, the CVMI eschews a dynamics for value states, so there's no specification for how likely it is that the apparatus will have a ready value state just before interaction with the particle. Thus, this proposal is unsuccessful.

The sort of interaction described above is not unusual, even by the lights of the CVMI. According to the CVMI the dynamic state never collapses, and hence the world in which we find ourselves is represented by one branch of a universal dynamic state. In some branch there might be 10^{25} particles forming an apparatus in a ready state, while in another branch those particles might be in a completely different state. Since the particles constituting an apparatus in one branch will presumably never be in an apparatus ready state in all of the branches, *according to the CVMI the everyday processes that we take to be measurements aren't*. Hence, according to the CVMI rules (b) and (c) do not apply to any of the everyday processes we take to be measurements.

Van Fraassen (1991, 233-7) proposes a model of state preparation which one could use to attempt a response to this argument against the CVMI. He suggests that there are some ways of preparing a system so that its final state is pure. Ruetsche (1996, S343-5) aptly criticizes van Fraassen's account and proposes, but does not endorse, a contrary account. The problem for the CVMI is that neither account presents a way of characterizing our actual state-preparation procedures; they just present special sorts of preparation procedures. Hence, neither account shows that our actual state-preparation procedures leave systems in pure states.

Ruetsche mentions this in a footnote: she says of her preparation procedure, (UP), that

> (UP) does not solve the Preparation Problem for MIs [Modal Interpretations]. That problem requires MIs to secure our actual preparation practices, and most of these practices do not conform to (UP). (1996, S343)

Ruetsche, however, does not draw the lesson that I do, which is that according to the CVMI all the everyday processes that we take to be measurements aren't.

I conclude that there is no hope of the CVMI being a successful complete theory – it makes no predictions (beyond rules (a) and (d)) for the outcomes of the everyday processes we take to be measurements. But the CVMI may still be worth thinking about, for two reasons.

(1) We can treat the CVMI as simply a theory that makes predictions for idealized textbook situations, where the initial state of the apparatus is a ready state. While this could be helpful, there are many other theories that can make the same idealized predictions, and can also make predictions about the interactions that occur in practice.

(2) We can modify the CVMI so that it does make predictions for the interactions that we take to be measurements. However, I don't know of any modification which would clearly solve the problem. A potential solution would be to add a dynamics for value states. There exists a standard dynamics for value states (half-jokingly called the Bohm-Bell-Vink-Bub-Clifton dynamics) which could be used. (See Bub (1997, 137-145) for details.) A problem with such a modification is that it would violate the desideratum that a system is assigned a definite value state only when required. A further modification can bring us closer to the desideratum. Call the state which evolves according to the Bohm-Bell-Vink-Bub-Clifton dynamics a 'pseudo-value state'. Let this state pick out a branch of the dynamic state of the universe. If a measurement occurs within that branch, then specify that at the end of the measurement the system actually has the value state given by the pseudo-value state. This interpretation is quite different than the original CVMI, but it may be the closest we can come to fulfilling the desiderata of the Copenhagen interpretation.

4. Conclusion. One of the main desiderata of the Copenhagen interpretation which the CVMI wants to respect is the view that a system is assigned a particular definite value state only when required. The CVMI attempts to follow this desideratum by assigning an observable a particular definite value in only two situations: when the system is in an eigenstate of the observable, and when the system is at the end of a measurement process. Ruetsche argues that the CVMI's characterization of measurement processes is too narrow; she points to certain processes that we take to be measurements but are not according to the CVMI. I've shown that this is not a problem for the CVMI, since such processes don't occur in practice. Nevertheless, I've shown that the spirit of Ruetsche's objection is correct: the CVMI's characterization of measurement processes is too narrow. None of the everyday processes that we take to be measurements is a measurement according to the CVMI, and hence the CVMI makes no predictions (beyond rules (a) and (d)) for such processes.

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