

# Einstein's Untimely Burial

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## Abstract

There seems to be a growing consensus that any interpretation of quantum mechanics other than an instrumentalist interpretation will have to abandon the requirement of Lorentz invariance, at least at the fundamental level, preserving at best Lorentz invariance of phenomena. In particular, it is often said that the collapse postulate is incompatible with the demands of relativity. It is the purpose of this paper to argue that such a conclusion is premature, and that a covariant account of collapse can be given according to which the state histories yielded by different reference frames are the Lorentz transforms of each other.

## 1 Introduction

There seems to be a growing consensus that any interpretation of quantum mechanics other than a instrumentalist one—that is, any interpretation that purports to provide a description of events and processes between measurements—will have to sacrifice the requirement of Lorentz invariance at the fundamental level,<sup>1</sup> preserving at best phenomenal Lorentz invariance, or invariance regarding empirical predictions (see, *e.g.*, Cushing 1994, Maudlin 1994, 1996). In particular, it has often been said that the collapse postulate is incompatible with relativity and hence that interpretations that take state-vector reduction to be a real physical process must abandon relativity, perhaps by introducing a preferred notion of distant simultaneity. It is the purpose of this paper to argue that such a conclusion is premature at best, and that there is reason to believe that an account of quantum-mechanical state reduction can be given that does not introduce a preferred foliation.

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<sup>1</sup>The term “fundamental Lorentz invariance” was introduced by Dickson and Clifton (1998).

It is not enough to formulate the theory in a manifestly covariant form. If there are differences in the accounts given with respect to different reference frames, then one must be able to argue, as Einstein did in formulating the special theory of relativity, that these differences are nothing more than different descriptions of the same reality, using different coordinates.

## 2 The Relativity of Entanglement

The notion of an instantaneous state of a spatially extended object, or an extended system of objects, is a foliation-relative notion. The state of the system is its state along some temporal slice, or, in other words, its state along some spacelike hyperplane or other spacelike hypersurface of simultaneity. This is vividly illustrated by the well-known example of the pole and the barn. With respect to a foliation of hyperplanes orthogonal to the barn's worldline, there is a temporal slice of the pole that lies entirely within the temporal slice of the barn along the same hyperplane. With respect to a foliation of hyperplanes orthogonal to the pole's worldline, there is no temporal slice of the pole that lies entirely within the temporal slice of the barn along the same hyperplane.

A comment is in order to forestall potential misunderstandings. Although, in special relativity, one associates a particular foliation of spacetime into spacelike hyperplanes with each state of inertial motion, which could be the state of motion of some observer, to say that something is foliation-relative is not to say that it is relative to an observer or that it is subjective in any way. No one is obliged to use his or her rest frame as a preferred frame of reference; quite the contrary, it is the lesson of relativity that it is quite immaterial which inertial reference frame one uses. Those who hold that an observer must always refer motion to that observer's rest frame, and hence must always regard him or herself as being at rest, are invited to consider whether it makes sense, while driving in a car, to look at the speedometer and say, "I'm moving at 50 miles per hour (with respect to the road)" rather than "The road is moving at 50 miles per hour (with respect to me)." A choice of reference frame is a choice of coordinate system; a choice of foliation is a choice of a global time coordinate, nothing more.

Let us now consider the transformation of quantum states. Consider two spin- $\frac{1}{2}$  particles, at rest with respect to some reference frame  $K$ . We will assume that the particles are localized in two regions, located at  $\mathbf{x}_1$  and  $\mathbf{x}_2$ , respectively, that are small enough, compared to the distances between them, that the regions may be regarded as points, but large enough compared to the Compton wavelength of the systems that the systems can indeed be regarded as localized within the regions. Let  $P$  be a point on  $S_1$ 's worldline, and let  $Q$  and  $R$  be two points on  $S_2$ 's worldline, with  $R$  in the past of  $Q$  (see Figure 1). Let  $s$  be a spacelike hypersurface that intersects  $S_1$ 's worldline at  $P$  and  $S_2$ 's worldline at  $R$ , and let  $s'$  be a spacelike hypersurface that also intersects  $S_1$ 's worldline at  $P$ , and intersects  $S_2$ 's worldline at  $Q$ .

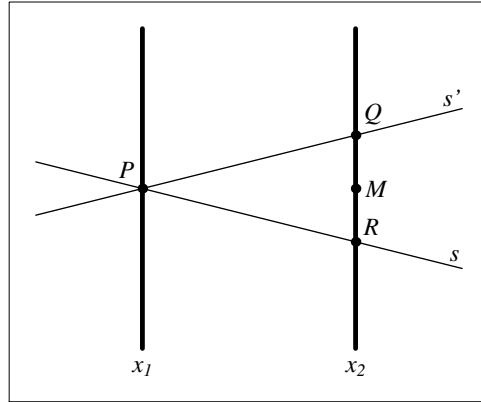


Figure 1.

Suppose, first, that the initial state of the combined system is

$$|\psi_1(0)\rangle = |z+\rangle_1 \otimes |z-\rangle_2, \quad (1)$$

and suppose that a measurement of spin- $x$  is performed on  $S_2$  at the spacetime point  $M$ , between  $R$  and  $Q$ . The result of such a measurement will be either  $+1$  or  $-1$ , and, by the projection postulate, the state of  $S_2$  after the measurement will be either  $|x+\rangle$  or  $|x-\rangle$ . Suppose that the outcome is  $+1$ . Then the state of the combined system along  $s$  is

$$|\psi_1(t_s)\rangle = |z+\rangle_1 \otimes |z-\rangle_2 \quad (2)$$

whereas the state of the system along  $s'$  is

$$|\psi'_1(t'_{s'})\rangle = |z'+\rangle_1 \otimes |x'+\rangle_2. \quad (3)$$

There is nothing paradoxical in this state of affairs; the difference in the two states is merely a consequence of a local change pertaining to  $S_2$ , plus the familiar relativity of simultaneity; the two hyperplanes link up two different moments in the evolution of  $S_2$  with the same moment in the evolution of  $S_1$  to form their respective instantaneous temporal slices of the combined system.

Now consider a scenario in which the initial state of the combined system is

$$|\psi_2(0)\rangle = |z-\rangle_1 \otimes |z+\rangle_2. \quad (4)$$

Suppose, again, that spin- $x$  is measured at  $M$ , and that again the result is  $+1$ . Then, along  $s$ , the state of the system is

$$|\psi_2(t_s)\rangle = |z-\rangle_1 \otimes |z+\rangle_2 \quad (5)$$

while the state of the combined system along  $s'$  is

$$|\psi'_2(t'_{s'})\rangle = |z'-\rangle_1 \otimes |x'+\rangle_2. \quad (6)$$

Now let the initial state of the combined system be a linear superposition of the states in the first two scenarios:

$$|\psi_3(0)\rangle = C_1 |z+\rangle_1 \otimes |z-\rangle_2 + C_2 |z-\rangle_1 \otimes |z+\rangle_2 \quad (7)$$

We will suppose once again that a measurement of spin- $x$  is performed at  $M$ , and that the outcome is  $+1$ . Then, in this scenario, the state at any time, and along any hyperplane, is simply a superposition of the states in the first two scenarios. Along  $s$  we have,

$$|\psi_3(t_s)\rangle = C_1 |z+\rangle_1 \otimes |z-\rangle_2 + C_2 |z-\rangle_1 \otimes |z+\rangle_2 \quad (8)$$

Along  $s'$ , the state is:<sup>2</sup>

$$\begin{aligned} |\psi_3(t_{s'})\rangle &= C_1 |z'+\rangle_1 \otimes ({}_2\langle x' + |z'-\rangle_2) |x'+\rangle_2 + C_2 |z'-\rangle_1 \otimes ({}_2\langle x' + |z'-\rangle_2) |x'+\rangle_2 \\ &= \frac{1}{\sqrt{2}} (C_1 |z'+\rangle_1 + C_2 |z'-\rangle_1) \otimes |x'+\rangle_2. \end{aligned} \quad (9)$$

Here we have a superposition of two states that undergo local changes in the transition from  $s$  to  $s'$ , pertaining only to  $S_2$ . This change in the state vector ought also to count as a local change pertaining to  $S_2$ . Here again the two hyperplanes merely link up two different moments in the evolution of  $S_2$  with the same moment in the evolution of  $S_1$  to form instantaneous temporal slices of the combined system; the only difference is that we have here to do with a superposition of such spliced states.

There is, of course, an important difference between the third scenario and the first two. In the first two scenarios, the state of the system is a factorizable state along both hyperplanes. In the third, the state of the combined system is an entangled state along  $s$  and a factorizable state along  $s'$ . This seems odd, as we think of the factorizable state (9) as attributing a definite spin state to  $S_1$ , whereas in the entangled state (8),  $S_1$  has no spin state of its own at all. An analogy might help: it is as if a marriage could be dissolved unilaterally by the declaration of one spouse. If a husband and wife are some distance apart at the moment that the wife declares the marriage dissolved, then the question of whether, at a given point on his worldline that is spacelike separated from the declaration, the husband is married or divorced, requires a choice of hypersurface of simultaneity.

This circumstance, that, at a spacetime point  $P$  a system may be part of an entangled state along one hypersurface passing through  $P$ , and part of a factorizable state along another hypersurface passing through  $P$ , may be called the *relativity of entanglement*. It is a consequence jointly of the relativity of simultaneity and of modelling collapse as a local change in the state vector.

### 3 Transformations between foliations

The instantaneous state of an extended system, classical or quantum, is defined along a spacelike hypersurface. Within a foliation, the transition from one hypersurface to another

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<sup>2</sup>In discussing collapse, it will be more convenient not to require the state vector to be normalized at all times.

is given by the dynamical evolution of the system. How are states defined hypersurfaces belonging to different foliations related to each other?

Consider two systems,<sup>3</sup>  $S_1$  and  $S_2$ , which at time  $t$  are located, with respect to some reference frame  $\Sigma$ , at positions  $\mathbf{x}_1(t)$  and  $\mathbf{x}_2(t)$ , respectively. Let  $\Sigma'$  be a reference frame moving with velocity  $v$  in the positive  $x$ -direction relative to  $\Sigma$ , and let the transformation from  $\Sigma$  to  $\Sigma'$  be given by the Lorentz boost,

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y & z' &= z \\ t' &= \gamma(t - vx/c^2), \end{aligned} \tag{10}$$

where  $\gamma = 1/\sqrt{1 - v^2/c^2}$ . Let  $\mathcal{H}_1$  and  $\mathcal{H}_2$  be the Hilbert spaces associated with  $S_1$  and  $S_2$ , respectively, and let the states of the two systems at time  $t$  be given, with respect to  $\Sigma$ , by  $|u(t)\rangle_1$  and  $|v(t)\rangle_2$ .

By Wigner's theorem (see Weinberg 1995, 91–96), there is a unitary transformation operator  $\Lambda_1$  that takes vectors in  $\mathcal{H}_1$  into their transforms under the Lorentz boost (10). Similarly, there is a unitary Lorentz boost operator  $\Lambda_2$  on  $\mathcal{H}_2$ . As Dickson and Clifton (1998, 15) have shown, the transformation operator on  $\mathcal{H}_1 \otimes \mathcal{H}_2$  is simply  $\Lambda_1 \otimes \Lambda_2$ . Therefore, for any time  $t$ , the Lorentz transform of  $|\psi(t)\rangle_{12}$  is

$$\begin{aligned} |\psi'\rangle_{12} &= \Lambda_1 |u(t)\rangle_1 \otimes \Lambda_2 |v(t)\rangle_2 \\ &= |u'(t'_1(t))\rangle_1 \otimes |v'(t'_2(t))\rangle_2, \end{aligned} \tag{11}$$

where

$$t'_i(t) = \gamma(t - v x_i(t)/c^2). \tag{12}$$

These are not instantaneous temporal slices of the combined system with respect to the  $\Sigma'$ 's hyperplanes of simultaneity. Rather, the transform of a state  $|u(t)\rangle_1 \otimes |v(t)\rangle_2$  at a given  $\Sigma$ -time  $t$  is a description of the states of the two component systems at two different times, as measured by  $\Sigma'$ 's time coordinate. In order to get instantaneous states with respect to  $\Sigma'$  from instantaneous states with respect to  $\Sigma$ , we need to know something of the dynamical evolution of the combined system.

Assume that the Hamiltonian of the combined system is simply the sum of the component Hamiltonians, with no interaction term,

$$H_{12} = H_1 \otimes I_2 + I_1 \otimes H_2. \tag{13}$$

Then the unitary evolution operator  $U(t; t_0) = e^{H(t-t_0)/i\hbar}$  factors:

$$U(t; t_0) = U_1(t; t_0) \otimes U_2(t; t_0). \tag{14}$$

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<sup>3</sup>The generalization to any finite number of systems is straightforward, and to continuous fields by a limiting process. See Tomonaga (1946).

Once we have a factorizable evolution operator, we can obtain states along the hyperplanes of constant  $t'$  from state defined on hyperplanes of constant  $t$ .

$$|u'(t')\rangle_1 \otimes |v'(t')\rangle_2 = \Lambda_1 U_1(t_1(t'); t) |u(t)\rangle \otimes \Lambda_2 U_2(t_2(t'); t) |v(t)\rangle, \quad (15)$$

where

$$t_i(t') = \gamma (t' + v x'_i(t')/c^2). \quad (16)$$

Because of the linearity of the Lorentz boost operators and of the evolution operators, the transformation (15) applies to any state of the combined system, and not only to factorizable states:

$$|\psi'(t')\rangle = \Lambda_1 U_1(t_1(t'); t) \otimes \Lambda_2 U_2(t_2(t'); t) |\psi(t)\rangle. \quad (17)$$

Thus, the state history of the combined system with respect to  $\Sigma'$  can be obtained from a state history given with respect to  $\Sigma$ , together with knowledge of the dynamics of the system.

Let us now add collapse to the picture,<sup>4</sup> and assume the existence of generalized evolution operators  $E_1(t; t_0)$ ,  $E_2(t; t_0)$  that approximate unitary evolution most of the time but produce a collapse whenever conditions are ripe (whatever that may turn out to be; such events ought not be confined to the laboratory). We can assume these operators to be linear if we don't require that they preserve the norm of the state vector. After a measurement of, say, spin- $x$  on  $S_1$ , the operator  $E_1(t; t_0)$  becomes either  $P^{|x^+}\rangle_1$  or  $P^{|x^-}\rangle_1$ , with the probability for each transition given by the usual quantum-mechanical rules for computing probabilities (note that these involve the *entire* global state vector; the probabilities are non-local quantities). Collapse evolution will *not* be assumed to be a reversible process, and so in general  $E_i(t; t_0)$  will be undefined for  $t < t_0$ . Because of this, we will not always be able to obtain a state of the system along any given hyperplane of constant  $t'$  from any state of constant  $t$ ; in order to obtain a  $\Sigma'$ -state at time  $t'$ , we must start with a  $\Sigma$ -state at a time  $t$  such that  $t \leq t_1(t')$  and  $t \leq t_2(t')$ . With this proviso, we have:

$$|\psi'(t')\rangle = \Lambda_1 E_1(t_1(t'); t) \otimes \Lambda_2 E_2(t_2(t'); t) |\psi(t)\rangle. \quad (18)$$

for  $t \leq \text{Min}[t_1(t'), t_2(t')]$ . It thus remains true that a complete state history given with respect to  $\Sigma$  uniquely determines the state history given with respect to  $\Sigma'$ .

## 4 The EPR-Bohm experiment

Let us now apply this picture of the collapse of foliation-relative states to the familiar EPR-Bohm experiment.

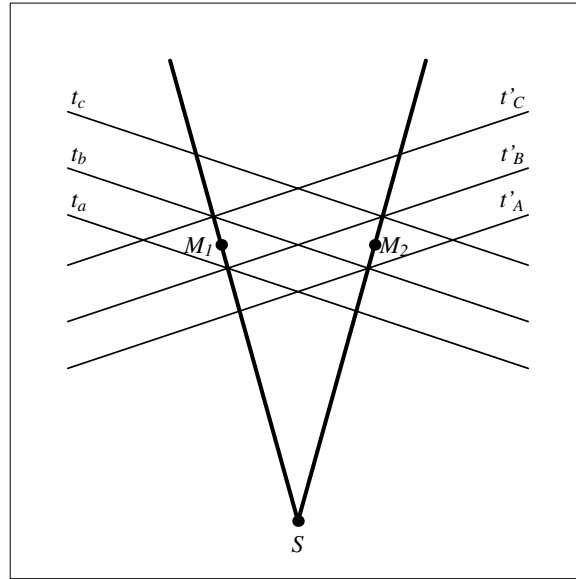
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<sup>4</sup>Though the notion of foliation-relative state evolution was present from the early days work on relativistic quantum theories (see Dirac 1933, Tomonaga 1946, Schwinger 1951), the application of this notion to state-vector collapse was perhaps first made by Aharonov and Albert (1984). Fleming (1986, 1989, 1996) has perhaps been the most prominent exponent of foliation-relative collapse.

Let  $S_1$  and  $S_2$  be spin- $\frac{1}{2}$  particles, initially in the singlet spin state,

$$\begin{aligned} |\psi(0)\rangle &= |z+\rangle_1 \otimes |z-\rangle_2 - |z-\rangle_1 \otimes |z+\rangle_2 \\ &= |x+\rangle_1 \otimes |x-\rangle_2 - |x-\rangle_1 \otimes |x+\rangle_2. \end{aligned} \quad (19)$$

Suppose that measurements of spin- $x$  and spin- $z$  are performed on  $S_1$  and  $S_2$ , respectively, at spacelike separation, and suppose that the outcomes of the two experiments are both +1. Let  $\Sigma$  be a reference frame with respect to which the measurement on  $S_1$  is performed first, and let  $\Sigma'$  be a reference frame with respect to which the order of the measurements is reversed. Let  $t_a$  be a time, with respect to  $\Sigma$ , prior to the measurement on  $S_1$ , let  $t_b$  be after the measurement on  $S_1$  but before the measurement on  $S_2$ , and let  $t_c$  be a time after both measurements are completed (see Figure 2).



**Figure 2. Spacetime diagram of the EPR-Bohm experiment.**

Define the ‘collapse evolution operators,’

$$E_1(t) = \begin{cases} I_1, & \text{before the measurement on } S_1. \\ P^{|x+\rangle_1}, & \text{after the measurement on } S_1. \end{cases} \quad (20)$$

$$E_2(t) = \begin{cases} I_2, & \text{before the measurement on } S_2. \\ P^{|z+\rangle_2}, & \text{after the measurement on } S_2. \end{cases}$$

The state history of the two-particle system, with respect to  $\Sigma$ , is given by

$$|\psi(t)\rangle = E_1(t) \otimes E_2(t) |\psi\rangle_{\text{singlet}}. \quad (21)$$

Along the hyperplane  $t = t_a$ , the state is

$$\begin{aligned} |\psi(t_a)\rangle &= E_1(t_a) \otimes E_2(t_a) |\psi\rangle_{singlet} \\ &= I_1 \otimes I_2 |\psi\rangle_{singlet} = |\psi\rangle_{singlet}. \end{aligned} \quad (22)$$

Along the hyperplane  $t = t_b$ , we have,

$$\begin{aligned} |\psi(t_b)\rangle &= E_1(t_b) \otimes E_2(t_b) |\psi\rangle_{singlet} \\ &= P^{|x+\rangle_1} \otimes I_2 |\psi\rangle_{singlet} \\ &= P^{|x+\rangle_1} |x+\rangle_1 \otimes |x-\rangle_2 - P^{|x-\rangle_1} |x+\rangle_1 \otimes |x+\rangle_2 \\ &= |x+\rangle_1 \otimes |x-\rangle_2. \end{aligned} \quad (23)$$

Along the hyperplane  $t = t_c$ , the state is,

$$\begin{aligned} |\psi(t_c)\rangle &= E_1(t_c) \otimes E_2(t_c) |\psi\rangle_{singlet} \\ &= P^{|x+\rangle_1} \otimes P^{|z+\rangle_2} |\psi\rangle_{singlet} \\ &= P^{|x+\rangle_1} |x+\rangle_1 \otimes P^{|z+\rangle_2} |x-\rangle_2 \\ &= -\frac{1}{\sqrt{2}} |x+\rangle_1 \otimes |z+\rangle_2. \end{aligned} \quad (24)$$

To describe the evolution of the state as given by  $\Sigma'$ , we define,

$$E'_1(t') = \begin{cases} I_1, & \text{before the measurement on } S_1. \\ P^{|x'+\rangle_1}, & \text{after the measurement on } S_1. \end{cases} \quad (25)$$

$$E'_2(t') = \begin{cases} I_2, & \text{before the measurement on } S_2. \\ P^{|z'+\rangle_2}, & \text{after the measurement on } S_2. \end{cases}$$

These operators are simply the Lorentz transforms of the operators (20).

Along the hyperplane  $t' = t'_A$ , the state is

$$\begin{aligned} |\psi'(t'_A)\rangle &= E'_1(t'_A) \otimes E'_2(t'_A) |\psi'\rangle_{singlet} \\ &= I_1 \otimes I_2 |\psi'\rangle_{singlet} = |\psi'\rangle_{singlet}. \end{aligned} \quad (26)$$

Along the hyperplane  $t' = t'_B$ , we have,

$$\begin{aligned} |\psi'(t'_B)\rangle &= E'_1(t'_B) \otimes E'_2(t'_B) |\psi'\rangle_{singlet} \\ &= I_1 \otimes P^{|z'+\rangle_2} |\psi'\rangle_{singlet} \\ &= |z'+\rangle_1 \otimes P^{|z'+\rangle_2} |z'-\rangle_2 - |z'-\rangle_1 \otimes P^{|z'+\rangle_2} |z'+\rangle_2 \\ &= -|z'-\rangle_1 \otimes |z'+\rangle_2. \end{aligned}$$

Along the hyperplane  $t' = t'_C$ , the state is

$$\begin{aligned}
|\psi'(t'_C)\rangle &= E'_1(t'_C) \otimes E'_2(t'_C) |\psi'\rangle_{singlet} \\
&= P^{|x'+\rangle}_1 \otimes P^{|z'+\rangle}_2 |\psi'\rangle_{singlet} \\
&= -P^{|x'+\rangle}_1 |z'+\rangle_1 \otimes P^{|z'+\rangle}_2 |z'+\rangle_2 \\
&= -\frac{1}{\sqrt{2}} |x'+\rangle_1 \otimes |z'+\rangle_2.
\end{aligned} \tag{27}$$

The difference in the two state histories can clearly be seen to arise solely from the difference in foliations used to define the instantaneous states of the evolving system, and therefore, the two state histories are, in a straightforward way, the Lorentz transformations of each other. This in spite of the fact that, along  $t_b$ , the state of the system is  $|x+\rangle_1 \otimes |x-\rangle_2$ , even though  $S_2$  is *never* in the state  $|x-\rangle_2$  on the state history according to  $\Sigma'$ , and, along  $t'_B$ , the state of the system is  $-|z-\rangle_1 \otimes |z+\rangle_2$ , even though, on the state history according to  $\Sigma$ ,  $S_1$  is never in the state  $|z'-\rangle$ .<sup>5</sup>

Since there is a collapse between the hyperplane  $t = t_b$  and the hyperplane  $t' = t'_B$  in both directions, we cannot apply (18) directly to one of these states to obtain the other. This doesn't mean that the two states are unrelated, however, as they both can be obtained from the state along  $t = t_a$ .<sup>6</sup>

$$|\psi(t_b)\rangle = E_1(t_b) \otimes E_2(t_b) |\psi(t_a)\rangle \tag{28}$$

$$|\psi(t'_B)\rangle = \Lambda_1 \otimes \Lambda_2 E_2(t_c) |\psi(t_a)\rangle \tag{29}$$

For convenience, the hyperplanes  $t = t_a$  and  $t' = t'_B$  have been taken to intersect  $S_1$ 's worldline at the same point, and the hyperplanes  $t = t_c$  and  $t' = t'_B$  have been taken to intersect  $S_2$ 's worldline at the same point.

## 5 Probabilities, causality, and passion-at-a-distance

The quantum-mechanical rule for assigning probabilities to outcomes of measurements is easily formulated in terms of foliation-relative states. Let  $F$  be any foliation, and let  $\sigma$  be the member of  $F$  passing through the measurement event (or, perhaps, immediately to the past of the measurement event). Then the expectation value of a measurement of an observable  $A$  is given by

$$\langle A \rangle_\sigma = \frac{\langle \psi(\sigma) | A | \psi(\sigma) \rangle}{\langle \psi(\sigma) | \psi(\sigma) \rangle}. \tag{30}$$

Probabilities, therefore, are foliation-relative. In spite of this, the rule (30) does not pick out a *preferred* foliation, as long as the Hamiltonian contains no nonlocal interaction terms.

<sup>5</sup>It should be pointed out that, whenever the states along two hypersurfaces passing through a point  $P$  on  $S_1$ 's worldline both assign a definite spin state to  $S_1$ , they will agree on what spin state it is.

<sup>6</sup>This answers an objection raised by Maudlin (1996, 302) to foliation-relative collapse.

Although a statistical test of quantum mechanics requires calculation of probabilities with respect to some foliation-relative state evolution, it doesn't matter which foliation is chosen.<sup>7</sup>

The foliation-relativity of probabilities has the odd consequence that a collapse event, even though it is a local event, will sometimes be assigned different probabilities by states along different hyperplanes passing through a single point. There is no limit on *how* different these probabilities can be. Suppose, for example, that the initial state of our two-particle system is, instead of the singlet state, the state,

$$\epsilon |z+\rangle_1 \otimes |z-\rangle_2 + \sqrt{1 - |\epsilon|^2} |z-\rangle_1 \otimes |z+\rangle_2 \quad (31)$$

with  $|\epsilon| \ll 1$ . Suppose that measurements of spin- $z$  are performed on both particles at spacelike separation, and that the outcome of the measurement on  $S_2$  is  $-1$ . Then, a hypersurface passing through the measurement on  $S_1$  and crossing  $S_2$ 's worldline to the past of the measurement on  $S_2$  will assign a very small probability  $|\epsilon|^2$  to the outcome  $+1$  of the measurement on  $S_1$ , whereas a hypersurface passing through the measurement on  $S_1$  and crossing  $S_2$ 's worldline to the future of the measurement on  $S_2$  will assign a probability 1 to the same outcome.

The fact that probability assignments do not factor into independent local probabilities (and, by Bell's theorem, cannot be regarded as supervening on independent local probabilities), is seen by some as a form of superluminal causal influence (*e.g.* Maudlin 1994). There is, however, a marked difference between the Bell-inequality violating dependence between systems in entangled states and an interaction of the sort that would be modelled by an interaction term in the Hamiltonian of the combined system. For spatially separated systems, such a term could not result in Lorentz-invariant state evolution, or even Lorentz-invariant statistics, but would on the contrary require a preferred notion of distant simultaneity. Indeed, any such interaction term will permit superluminal signalling via suitable measurements on suitable states.

It is not a matter of great moment whether we call the quantum-mechanical failure of probabilistic independence of spacelike separated events a form of causal influence, or invent a new term for it, such as "passion-at-a-distance" (Shimony 1984). Brian Skyrms (1984) is correct when he says that our notion of causality is an "amiably confused jumble" which unravels when applied to the quantum domain. What does matter is that we not ignore the fact that there is a difference between the way in which quantum mechanics treats this sort of influence and its treatment of causal interactions modelled by interaction terms in the Hamiltonian; if our physical theories are a guide for forming causal notions and applying them to the world, this suggests that at the very least we are dealing with something other than familiar causal interactions. Moreover, there seems to be a physical basis for this distinction, as exhibited by the fact that interactions given by interaction terms in the Hamiltonian can be manipulated for signalling, whereas the lack of probabilistic independence that arises

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<sup>7</sup>This suggests that there ought to be a way to formulate the rule in a manifestly covariant way, without mention of any non-intrinsic structures; so far, however, this has not been done. Unless and until such a formulation is available, we must refer probability assignments to foliation-relative states.

from quantum entanglement cannot, in the absence of nonlocal interaction terms in the Hamiltonian, be used to send superluminal signals.<sup>8</sup>

The distinction is important because, relativity theory does not, by itself, prohibit superluminal causation. If the special theory of relativity gives us reason to disbelieve in superluminal causation, the reason lies in our belief that the causal relation requires a unique temporal ordering of cause and effect. What is ruled out by relativity is any relation between spacelike separated events that requires a unique temporal ordering. The relation of signal transmission to reception is such a relation. It appears that the relation that holds between quantum systems in an entangled state is not such a relation. If this is so, then there is reason to hope for peaceful coexistence between quantum mechanics and special relativity after all.

## 6 Objections

The picture presented here is of stochastic evolution of foliation-relative states, with no foliation preferred. A number of objections have appeared in the literature to such a notion. There is not space here to address these objections in full, but it is possible to give the outlines of a response to the chief objections.

One objection is that foliation-relative state evolution, and the associated notion of foliation-relative becoming, makes such evolution a subjective matter and results in a breakdown of intersubjectivity (Dorato 1995, 593). A preliminary answer to this objection is to point out that the objection presupposes that a foliation is the extended present of some actual or possible observer; the special theory of relativity does not commit us to this. A choice of foliation is a matter of choice of coordinates, rather than a matter of the state of motion of the observer. A fuller answer to this question would have to address the larger issue of which it is a part, which is the issue of whether the foliation-relative notion of becoming that goes along with foliation-relative state evolution suffices for a realist, probabilistic interpretation of quantum mechanics (see also Maxwell 1985, Saunders 1996). I believe that it does, but I cannot do justice to the issue within the scope of this paper.

A second objection, due to Maudlin, stems from a supposed ontological independence of state histories defined along different foliations. Maudlin, reacting primarily to Fleming (1989), wrote,

The wave function on a hyperplane is, as it were, ontologically atomic. Wave functions defined on hyperplanes in the same family can be related by a dynamics which uses the family as a substitute for absolute time, but relations among wave functions from different families are obscure at best (1996, 302).

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<sup>8</sup>This point has, of course, been made many times before, and is often couched in terms of Jon Jarrett's factorization of the Bell locality condition into a conjunction of the condition that Jarrett (1984) calls "Locality" and Shimony (1986) calls "Parameter Independence," on the one hand, and the condition that Jarrett calls "Completeness" and Shimony calls "Outcome Independence," on the other.

If this were correct, then a dilemma would ensue. Either the state histories along each foliation would agree as to the macroscopic outcomes of experiments, or they wouldn't. If all foliations agreed on the outcome of experiments, this would be a coincidence inexplicable on the basis of the dynamics of the theory, which only connect states *within* a foliation (Maudlin 1996, 301). If, on the other hand, the state histories did not agree on such outcomes, “[t]he wave functions on each family of hyperplanes would then completely decouple, yielding an independent world for each foliation” (302).

The alleged ontological independence of state histories along differing foliations does not exist, however.<sup>9</sup> Different foliations are merely different ways of dividing spacetime into 3-dimensional spaces of simultaneity, and accounts given with respect to different foliations are merely different accounts of the same processes and events. Although one cannot always transform directly from the state on one hypersurface to a state on another, a state history given with respect to one foliation determines the state history with respect to any other foliation. The relations among state vectors from different families are given by our equation (18) and its generalizations.

A third objection, also due to Maudlin, is that the relativity of entanglement “shocks intuitions which are formed by acquaintance both with Relativity and with non-relativistic quantum mechanics” (1994, 209). Although, when a system is entangled with another, this is clearly a relation between the two systems (and hence it would not, perhaps, be surprising if the nature of this entanglement were a foliation-relative affair), when a system is *not* entangled with another, it seems plausible to assume that this is a matter purely of the intrinsic state of it and hence ought to be an invariant fact about the system. Acceptance of collapse as part of foliation-relative state evolution requires regarding this intuition, formed, as Maudlin points out, by exposure to relativity and *non-relativistic* quantum mechanics, as mistaken. To go back to our anthropocentric analogy: if marriage is a relation, so too is divorce! It would be interesting to explore whether the manner in which the relativity of simultaneity shocks our intuitions shares any important similarities with the counterintuitive features of relativistic quantum theory without collapse.

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<sup>9</sup>That is, it does not exist on the interpretation advocated here. It is not my intention to examine whether Maudlin has correctly interpreted Fleming. The issue at hand is whether the objection succeeds against the interpretation advocated in this paper.

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